Risk, Return and Portfolio Allocation under Alternative Pension Systems with Incomplete and Imperfect Financial Markets[1]

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Abstract:

This paper uses stochastic simulations on a calibrated model to assess the impact of different pension reform strategies where financial markets are less than perfect. We investigate the optimal split between funded and unfunded systems when there are sources of uninsurable risk that are allocated in different ways by different types of pension system when there are imperfections in financial markets. This paper calculates the expected welfare of agents of different cohorts under various policy scenarios. We estimate how the optimal level of unfunded, state pensions depends on rate of return and income risks and also upon preferences.

1. Introduction:

The old age dependency ratio in nearly all developed economies (the ratio of those of pensionable age to those of working age) will be substantially higher in the future; in many cases (Germany, Italy, Japan) the ratio is likely to double. If unfunded (pay-as-you-go), state pensions are to continue to provide a large part of retirement incomes, then contribution rates in most countries will have to be substantially higher to balance the system. The desirability of providing a significant proportion of retirement income from unfunded pensions is therefore a key policy issue. It has generated a large literature on the reform of pension systems (see, for example, Feldstein (1996); Feldstein and Samwick (1998); OECD (1996); Mitchell and Zeldes (1996); Disney (1996); Kotlikoff (1996); Huang, Imrohoroglu, and Sargent (1997);

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Miles and Timmerman (1999); Sinn (1999) and Campbell and Feldstein (2001)). If unfunded pensions have substantial advantages then it might be worth paying the costs of higher contribution rates to preserve them. But if greater reliance on other sources of pension income, most obviously income from funded pensions (or more generally from private saving), can replace unfunded pensions without adverse effects (for example on the allocation of risk), then there would be associated long-run benefits of a higher stock of financial assets and lower, potentially less distortionary, labour taxes. But funded and unfunded pension systems allocate risk in different ways, so any analysis of the implications of different degrees of reliance on funded and unfunded pensions has to consider the welfare implications of different risk allocation mechanisms. It also has to address transitional issues – how does one engineer the move towards a system with a different degree of reliance upon funded pensions given existing pension obligations?

The central policy issues we address in this paper are ones which are relevant in all economies: what is the desirable split between funded and unfunded systems when there are sources of uninsurable risk that affect risk averse agents and where those risks are allocated in different ways by different types of pension system? How does reform re-allocate resources between generations on a transition where pension arrangements and demographics are changing? How does the distribution of welfare evolve both within and between different generations?

In this paper we use a model which we chose to calibrate to the Japanese economy to assess the issues. Japan is an important case for at least three reasons: 1. it is the second largest economy in the world; 2. ageing will be rapid; 3. it currently has a relatively generous, largely unfunded, state-run pension system. The pension arrangements in Japan are not dissimilar to those in many European countries and the likely trends in demographics are comparable, though on average more extreme. In our model we allow for the impact of changing demographics and focus not just on steady states. We also allow for less than perfect risk sharing opportunities: idiosyncratic risk is significant and cannot be fully insured against; longevity risk exists in an environment where although annuity type contracts may be available, the transactions costs in that market may be substantial. Returns on some financial assets are risky.

There already exists a substantial literature that uses calibrated models to address issues of risk allocation in a world with less than perfect risk sharing opportunities. Much of that literature focuses on the US economy – Hubbard and Judd (1987); Imrohoroglu and Imrohoroglu and Joines (1995); Storesletten, Telmer and Yaron (1999); and Campbell, Cocco, Gomes and Maenhout (2001) present results from models calibrated to the US economy. De Nardi, Imrohoroglu and Sargent (1999) and Kotlikoff, Smetters and Walliser (1999) look specifically at the transitional and long run impacts of phasing out unfunded, state pensions in the US. They find substantial long run welfare gains but non-trivial welfare losses for some generations on the transition path. One of the aims of this paper is to see whether results that hold for the US carry over to the case of Japan – a country facing a process of ageing that is more rapid and which poses more problems for the design of pension systems than is the case in the USA. Japan is also a smaller and more open economy than the US and rates of return on assets may therefore be less affected by domestic saving than in the US. Japan is therefore more representative of many countries in Continental Europe.

Any model that wants to say something useful about risk and uncertainty must take account of several factors:

1. that individuals face substantial, largely idiosyncratic, risks that affect their labour income and are unlikely to be able to insure against such risks;
2. that borrowing against human capital is difficult, if not impossible;
3. that returns on most financial assets are volatile and uncertain;
4. that to the extent that individuals depend upon their own accumulated funds for retirement resources the way in which annuities markets work may be important (state run, unfunded systems will be unaffected by the efficiency of annuities markets because the government is effectively providing insurance itself);
5. if it is to be useful for policy purposes the model should consider the transition from one policy regime to another.

Solving models with all these features is difficult. Imrohoroglu, Imrohoroglu and Joines (1995, 1999a) investigate the role of social security in a general equilibrium setting with labour income uncertainty but non-stochastic rates of return and no annuities. Huang, Imrohoroglu and Sargent (1997) focus on the intergenerational impact of various social security systems on transition paths. They allow for stochastic labour income but there is no uncertainty on rates of return. Hubbard and Judd (1987) focus on the impact of credit restrictions, but in a model with no uncertainty about earnings or rates of return. Storesletten, Telmer and Yaron (1999) focus on the risk sharing implications of alternative social security systems; they concentrate on stationary states in a model with no uncertainty about rates of return. (See Imrohoroglu, Imrohoroglu and Joines (1999b) for an excellent survey on computational models of social security)).

Campbell, Cocco, Gomes and Maenhout (2001) consider the portfolio allocation implications of various pension arrangements. They consider the long-run pattern of lifetime savings and portfolio allocation in the presence of income and rate of return uncertainty and with various pension arrangements. But they do not model transitions from one pension regime to another. De Nardi, Imrohoroglu and Sargent (1999) and Kotlikoff, Smetters and Walliser (1999) both model transitions with an overlapping generations, general equilibrium model. But we also allow for both idiosyncratic (income) shocks and rate of return uncertainty.

The central policy issue we address is what is the optimal degree of reliance upon private saving. The private saving vehicles that we consider can be thought of as flexible personal retirement accounts – that is defined contribution (DC) personal pensions. Personal funded pensions may allow people to insure perfectly against some risks – *if* annuities are available then length of life risk can be avoided, but the terms on which that insurance is available will depend on the costs for financial firms of offering contracts that ensure an acceptable return. But personal (DC) pensions mean that labour income risk from working years, which will have an impact on the contributions to a personal pension fund, have lasting effects upon pension income; such pensions obviously also generate rate of return risk. Given this we consider what role might be played by unfunded, state pensions that give varying degrees of insurance against labour income risk and are not dependent on rate of return risk. We take into account shifting demographics that alter the contribution rate needed to balance an unfunded, state run system.

In calculating the optimal behavior of individuals we take account of all the sources of risk we noted above were essential. In particular, we assume that individuals face random shocks to labour income throughout their working life; some of these shocks are transitory but some are highly persistent. We also assume that they face uncertainty about the returns they will earn on at least some sorts of financial assets. We assume that there is a safe asset but there are also risky assets which, on average, earn higher returns. We assume individuals are risk averse and that they understand the risks of investing in different sorts of assets and are also aware of the uncertainty over how long they will live. We then use numerical techniques to calculate optimal profiles of consumption, saving and portfolio allocation for individuals over their lives. We aggregate these decisions to construct the macroeconomic aggregates and also construct measures of welfare. Solving this sort of model is difficult and we use numerical techniques to work out optimal consumption-investment decisions.

We set the critical parameters in the model (parameters of the utility functions such as degrees of risk aversion and rates of time preference, and characteristics of the labour income profile over life) by reference to recent data from the Japanese economy. We simulate the model using different settings for the key policy variables. We are able to show how different degrees of generosity of unfunded pensions affect the evolution of overall saving rates, levels of national income, and the allocation of savings across different assets. We are also able to make welfare comparisons. We construct a measure of welfare by estimating the expected utility of individuals born at various times. We can calculate the average gains and losses for people of different ages of various reform strategies. We are also able to calculate the distribution of gains and losses to agents of a particular cohort.

Our results indicate several things.

I. The aggregate stock of financial assets, the level of national income and overall portfolio allocation are very sensitive to differences in the generosity of unfunded pensions. In the long run the aggregate stock of wealth might be about 50% higher if unfunded pensions were, on average, worth only around 40 percent of net of contributions average earnings, as opposed to around 70 percent. Completely phasing out state pensions would in the long run roughly double the aggregate stock of wealth.

II. Long-run benefits to future generations of Japanese of a move towards greater reliance upon funded pensions are likely to be substantial; losses to the current generation of workers are smaller but not insignificant.

III. How much of financial wealth is invested in risky assets is very sensitive to both the level of state pensions and, less so, to the efficiency of financial markets. Even with quite low risk aversion (a coefficient of relative aversion of 3) we can explain substantial holdings of safe assets (of around 40%) if state pensions are very low. We do not need to assume extreme risk aversion or fixed costs of investing in risky assets to generate substantial investment in safe assets. This is so even though we use common assumptions about risk premia and the volatility of risky assets.

IV. Credit restrictions affect the answers substantially. Individuals find it difficult to borrow against future labour income (that is their human capital) and therefore any model with uncertainty over income and over length of life is one in which individuals naturally face borrowing constraints. We find that these constraints are likely to matter significantly. We also find that how serious borrowing constraints are, particularly amongst the elderly, depends very much on the pension environment.

V. A key finding is that longer run gains from a switch towards greater reliance upon funding, and away from an unfunded system where pensions are linked to salaries, do not just go to the better off. Many share in the gains, though those at the bottom of the income distribution do least well.

2. The model:

Given exogenous stochastic processes for labour income and for rates of return (and conditional on current and future pensions arrangements and mortality rates) agents choose consumption (and therefore saving) and portfolio allocation in each period to maximise expected lifetime utility. At the individual level we assume an additively separable form of the agent's lifetime utility function. We also assume a constant coefficient of risk aversion, the inverse of the intertemporal substitution elasticity. Agents are assumed to know the probabilities of surviving to given ages. Agent k who is aged j at time t maximises:

$$U_k = E_t \left[\sum_{i=0}^{i=T-j} s_{ij} \{ [c_{kt+i}]^{1-\zeta} / (1-\zeta) \} / (1+\rho)^i \right] \quad (1)$$

where T is the maximum length of life possible and the probability of surviving i more periods conditional on reaching age j is s_{ij}. ($s_{0j} = 1$). ρ is the rate of pure time preference; c_{kt+i} is consumption of the agent in period t+i.

ζ is the coefficient of relative risk aversion.

Agents face two constraints:

First there is a budget constraint governing the evolution of financial assets taken from one period to the next.

$$W_{k\,t+1} = [W_{k\,t} + \exp(y_{kt}).(1-\tau) - c_{kt} + b_{kt}].(\lambda \exp(r_{st}) + (1-\lambda)\exp(r_f)) \quad (2)$$

$W_{k\,t}$ is the stock of wealth of agent k in period t

y_{kt} is the log of gross labour income

τ is the proportional rate of the payroll tax (or contribution rate) on labour income. This rate is set to finance state pension expenditure.

b_{kt} is the level of the unfunded, state pension received by an agent

λ is the proportion of financial assets invested in risky assets

r_{st} is the one period (log) rate of return on risky financial assets held between period t and period t+1 This is stochastic.

r_f is the constant one period (log) rate of return on safe financial assets

We assume independent, normally distributed shocks affect the log returns on risky assets and a different set of idiosyncratic shocks affect log incomes; it is therefore natural to use log returns and log incomes in (2).

For ease of notation we have not given agent-specific subscripts to asset returns. But we will allow for the possibility that there are financial contracts available whose one period returns depends on whether an agent survives and where the rate of return would therefore differ across agents. We will describe shortly how rates of return on financial investments are determined.

Agents also face a borrowing constraint; wealth cannot be negative:

$W_{kt} \geq 0$ for all k and t.

This constraint may bind in various periods

We assume agents cannot take short positions in either safe or risky assets:

$0 \leq \lambda \leq 1$

In the model agents born before 1961 work from age 20 to the end of their 63rd year (if they survive that long) and are retired thereafter. Agents supply one unit of labour each year until (exogenous) retirement. Following plans for later retirement recently introduced by the Japanese government we allow for the retirement age to move up so that by 2030 retirement comes at age 65[4]. We assume that the profile of gross of tax labour income reflects three factors. First, there is a time-related rise in general labour productivity. Second, there is an age-related element to the growth of labour income over an agent's life. This is modelled as a quadratic in age.

The age-specific part of the log of labour income is:

$$\alpha + \gamma age - \theta age^2 \tag{3}$$

We set γ and θ so that the age-income profile matches patterns that have been typical in Japan (we discuss calibration issues in detail in the next section).

There are also idiosyncratic (agent specific) stochastic elements of labour income. The log of labour income for an agent is the sum of the age-related element, the time related element (part of which is a function of the aggregate capital to labour ratio) and the additive income shock. The income shock has a transitory component (ω) and a persistent component (u). Denoting the log of gross labour income of agent k who is aged j in period t as y_{kt} we have:

$$y_{kt} = \ln(\psi_t) + gt + \gamma.j - \theta.j^2 + u_{kt} + \omega_{kt} \tag{4}$$

$$u_{kt} = \phi\, u_{kt-1} + e_{kt}$$

[4] For a useful survey of the Japanese pension system see Clark and Mitchell (2002).

where $e \sim N(0, \sigma_e)$; $\omega \sim N(0, \sigma_\omega)$ and e and ω are iid and uncorrelated.

ψ_t is a common factor across agents which reflects the compensation per effective unit of labour at time t .

g is the rate of growth of labour productivity over time.

ϕ reflects the degree of persistence in the non-transitory idiosyncratic shocks to labour income; empirical evidence from a range of countries suggests ϕ is high and that idiosyncratic shocks to income typically have a high degree of persistence (We discuss the exact calibration of the model in Section IV).

We assume that rates of return on risky financial wealth vary across periods due to random shocks that hit stock and bond markets. Mean returns on risky assets and the excess of that mean over the safe rate (ie the risk premium) are given. Both rates of return and compensation per effective hour worked are independent of domestic saving. As a result of this open economy assumption there is potentially a divergence between the stock of wealth desired by agents and the capital stock used in production. For Japan, which has consistently run large current account surpluses for several decades, this divergence has indeed been substantial. National income and domestic production are not equal – we focus on national income as the measure of total resources created. This is the sum of aggregate labour incomes and the return on aggregate net wealth.

Financial assets:

Agents choose between safe and risky assets. In one version of the model we assume there are no annuities. But if annuities contracts do exist then agents with no bequest motive will buy them, even if because of the costs of offering such contracts (eg the costs to financial firms of screening) the rates of return they offer fall well short of those that could be offered at actuarially fair rates.

When we do allow for the existence of annuities then financial institutions take into account the probabilities of death of agents and offer age-related investment products with annuity features. We do this in a highly stylized way. If financial institutions could costlessly offer actuarially fair contracts then for every $ invested in period t, with a given risky/safe split, then conditional on survival the investor receives the market return adjusted for a probability of survival to the next period. Financial institutions effectively share out the savings of those who die across the survivors. If contracts could be offered at no transactions cost the probability used in making this adjustment is the true survival probability – in which case no resources are used up by financial intermediaries and all the proceeds of the estates of the dead go to survivors. But it is unrealistic to assume that savings contracts could be offered on those terms. So we allow for rates of return to be much lower than they would be with no transactions costs and actuarial fairness. In other words the money's worth ratio is less than unity[5]. In fact we will concentrate on two cases: first where the money's worth ratio is about what we see in the US and the UK where annuities markets are well developed and second where annuities markets are effectively absent (or so costly to run that agents just get the raw return on the assets with no uplift for a less than certain survival)[6]. In this latter case we are assuming that costs of offering contracts are such that substantial resources are used up in intermediation.

More formally here is how we model asset returns. *If* there is no annuity element (or survival uplift) to the return on assets the ex-post rate of return on a $ invested in the risky asset during period t by an agent k who is aged j and who survives to the next period is just given by:

$$\exp(r_{st}) \tag{5}$$

We assume r_s is the sum of the (time varying) mean log return and an unpredictable shock.

$r_{st} = r_\mu + v_t$

r_μ is the mean rate of return on risky assets

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[5] The money's worth ratio is the ratio between the expected payout on an annuity contract using actuarial survival probabilities and the cost of the annuity. With no transactions costs and competitive pricing it would be unity.

[6] Demand for annuities currently across OECD countries is limited. This reflects the generosity of state pensions (which are effectively annuities), imperfections in financial markets and, perhaps, bequest motives. We do consider the first two factors in our model but, as noted already, we follow Campbell et al (2001) in abstracting from bequest issues. Horioka (2001) presents evidence that the bequest motive in Japan is not strong.

v_t is the random element of the rate of return on assets in period t.
We assume v is iid and normal: $v \sim N(0, \sigma_r)$

For a $ invested in the safe asset the return to an agent aged j is:

$$\exp(r_f) \quad (6)$$

r_f is the safe asset at time t

If the annuities market is not entirely absent there may be some element of the return on assets that reflects the mortality risk of an agent. We model this in a highly stylized way and introduce a parameter, β, that reflects the costs of offering contracts with an annuities element. When β is 1 contracts are actuarially fair and there are no transactions costs; the money's worth ratio is unity. When β = 0 savings contracts are least actuarially fair. The survival probability implicit in the contract offered by a financial institution is a weighted average of the true survival probability and unity. β is the weight placed on the actuarially fair survival probability. When β is less than unity we interpret this as reflecting the costs of offering contracts. We can think of these as costs incurred by financial institutions in assessing the true survival probabilities of agents (eg. checking on age and health). The higher is β the more efficient are financial contracts[7].

Let the one period survival probability of an agent aged j be denoted s_{1j}
Then the generalizations of equations (5) and (6) for the ex-post return on risky and safe investments for an agent aged j who survives to the next period when the level of financial contract efficiency is β are:

$$\{\exp[r_\mu + v_t]\} / [\beta s_{1j} + (1-\beta)] \quad (7)$$

$$\{\exp[r_f]\} / [\beta s_{1j} + (1-\beta)] \quad (8)$$

This way of modeling the efficiency of annuity contracts allows the departure from actuarially fair contracts to vary with age. The greater is age, the lower the probability of surviving and for all β < 1 the greater is the departure from actuarially fair contracts. Recent empirical evidence from the US suggests that annuity rates do become increasingly less favorable with age. Mitchell, Poterba and Warshawsky (1999) estimate that the average US annuity in 1995 delivered payouts with expected present value of between 80% and 85% of each $ annuity premium for 65 year olds; but the payout ratio was less for older people. A payout ratio of 80% of the actuarially fair value for a 65 year old corresponds in our simulations (based on Japanese mortality rates[8]) to a value of β of about 0.3 if the rate of return on assets is a flat 6%. Friedman and Warshawsky (1988) report US payout ratios from the 1970's and 1980's of around 75% which corresponds to a β for current Japanese life expectancies of about 0.2. Brown, Mitchell and Poterba (1999) provide some evidence that in the UK annuities average about 90% of the actuarially fair rates. This corresponds to a β of around 0.55. In Japan there is less evidence on annuity market efficiency and a less deep market. The depth of the market is linked to the generosity of state pensions. The issue of what assumption to make about what annuity market efficiency will be with different pension arrangements is important. Our strategy is to consider two values for efficiency – a value that broadly corresponds to the latest estimates of the efficiency in the UK and US (β=0.5) and zero.

We allow for the portfolio weight on safe and risky assets to be chosen by agents at each period in their life. Bringing together all these elements we can finally write the ex-post rate of return paid on the savings of an agent aged j at time t who invests λ in the risky asset and (1-λ) in the safe asset:

$$\{\lambda \exp[r_\mu + v_t] + (1-\lambda)\exp[r_f]\} / [\beta s_{1j} + (1-\beta)] \qquad (9)$$

where β either equals 0 or 0.5.

State pensions

In Japan (as in many developed economies) the state run, PAYGO pension has a flat rate element and an earnings related element. For a typical Japanese worker the flat

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[7] For evidence of the degree of divergence from actuarial fairness in annuities contracts in the UK and the US see Friedman and Warshawsky (1988); Mitchell, Poterba and Warshawsky (1997) and Brown, Mitchell and Poterba (1999)

[8] using official Japanese estimated mortality rates.

rate pension currently generates around 1/3 of pension benefits and 2/3 come from a salary related pension. We use these ratios (1/3 flat rate; 2/3 salary related) and model the earnings related element as depending on final salary. The system is financed by the proportional tax on labour income levied on all those working. The tax rate is set to balance the unfunded state pension system in every period. We define the average replacement rate of the state pension as the ratio between the average pension paid in period t to someone just retired and the average gross income of those in the last year of their working life at period t-1. Pensions paid at retirement are therefore linked to movements in wages, but pensions subsequently are fixed in real terms.

The tax rate to finance state pensions of a given generosity is proportional to the replacement rate of the unfunded system. The factor of proportionality reflects the support ratio which in Japan is likely to change sharply over the next few decades. We are able to calculate the path of the balancing tax rate (τ_t) quite easily. Once we set the average replacement rates and given the exogenous demographics and labour supply – all of which are set independently of the random shocks - the balancing contribution rate can be calculated. That rate depends only upon the ratio of average state pensions to average earnings and the ratio between the number of people in receipt of pensions and the number of those working. Although there is uncertainty at the household level about income and about length of life, in aggregate the structure of the population and of average wages and pensions is known (by the law of large numbers). This means that the equilibrium contribution rate can be calculated independently of the realization of idiosyncratic income shocks; it is also independent of the realization of shocks to financial asset returns since this is not a funded system and pensions reflect only wages.

3. Solving the Model:

First order conditions:

The set of first order conditions from individual k's optimisation problem are:

if $c_{kt} < [W_{kt} + \exp(y_{kt}).(1-\tau_t) + b_{kt}]$

then (10)

$$U`(c_{kt}) = E_t [s_{1j} \{ U`[c_{kt+1}].\{ \lambda \exp(r_\mu + v_t) + (1-\lambda)\exp(r_f) \}/ [\beta s_{1j} + (1-\beta)] \} / (1+\rho)]$$

else:

$c_{kt} = [W_{kt} + \exp(y_{kt}).(1-\tau_t) + b_{kt}]$

and (11)

$U`(c_{kt}) \geq E_t [s_{1j} \{ U`[c_{kt+1}]. \{ \lambda \exp(r_\mu + v_t) + (1-\lambda)\exp(r_f) \} / [\beta s_{1j} + (1-\beta)] \} / (1+\rho)]$

where $U`(c_{kt})$ is $\partial U_k / \partial c_{kt}$

We also require a condition for optimal portfolio allocation:

Either :

$0 = E_t [U`[c_{kt+1}].\{ \exp(r_\mu + v_t) - \exp(r_f) \}]$

and $0 \leq \lambda \leq 1$

or

$0 < E_t [U`[c_{kt+1}].\{ \exp(r_\mu + v_t) - \exp(r_f) \}]$

and $\lambda = 1$ (12)

else

$0 > E_t [U`[c_{kt+1}].\{ \exp(r_\mu + v_t) - \exp(r_f) \}]$

and $\lambda = 0$.

(10) holds when the borrowing constraint is not binding. When the constraint binds complementary slackness implies that (11) holds. (12) is a standard condition for optimal portfolio allocation. Corner solutions may arise where agents wish to only invest in the safe asset or in the risky asset; for an internal solution the first equality at (12) must hold.

Solution technique:

Although characterising optimal plans is easy enough solving explicitly for optimal consumption and for the optimal accumulation path for funds is not possible. Instead we have to turn to numerical methods. We solve the problem backwards in a now standard way (see Deaton 1991, Zeldes 1989, and Hubbard et al. 1995.) This involves constructing large grids in the state space and solving for optimal saving and portfolio allocation at those grid points using numerical integration and interpolation. The state variables for an agent are age, the stock of wealth and the level of labour income.

At each point in the state space – ie points on the grid in ages, levels of wealth and levels of incomes – we need to solve the first order conditions (10) – (12). This involves finding the levels of consumption and portfolio allocations that solve the conditions given the two sources of uncertainty – rate of return shocks and idiosyncratic shocks to incomes. Both of those shocks are normally distributed. In calculating the expectations in (10) and (12) we have to use numerical integration .

Once we have calculated a solution grid we then generate life histories for cohorts each of size 100,000 households born at different times. To do that we create a set of paths for the idiosyncratic shocks to income for each member of every cohort. We then work out the optimal consumption-saving-portfolio allocation profile for every agent. Then we aggregate the decisions made by all cohorts alive at each date, taking account of the relative numbers of agents of each age at each point (based on official central demographic projections for Japan). We aggregate the saving and labour supply of all agents alive at each time and construct time series for the aggregate variables of interest. National income is the sum of labour income across all workers plus income earned on aggregate wealth.

What we end up with is 100,000 profiles of consumption and wealth for each cohort born at different years between 1900 and 2100. Each profile is different because it depends upon the particular path of realizations of shocks to income. Each path also depends upon the realizations of shocks to rates of return. In calculating optimal rules people base their decisions on the true ex-ante distribution of these shocks – which is normal. The realizations of rate of return shocks, which unlike the income shocks are common to all agents at a particular date, affect decisions.

The stochastic variables in the model are assumed to have a normal distribution (rates of return and the idiosyncratic shock to labour income come from independent normal distributions). In solving the individual optimisation problem we explicitly allow for individuals to form expectations based on those distributions. So at a particular point in time every household makes a decision, based on the state variables at that time (which are its level of wages and the value of financial assets) in such a way that the Euler equation is satisfied and expectations are based on tomorrow's random variables coming from a normal distribution. We then have a set of rules for every household which shows what it is optimal to consume and to save, and how portfolio allocation should be done, at every possible level of the state variables (or, more accurately, at a very large number of points in the state space since this is a numerical approximation). When we come to do simulations, we then need to show what would happen under a particular realisation of the random variables. Of course, any particular realisation of the random variables is one which, ex ante, has negligible probability. What is useful, nonetheless, is to show what would happen under particular realisations. We could show many different realisations of random variables. We choose to show a particular set. In one particular set of realisations for the aggregate uncertainty shock (that is the shock that affects the rates of return on risky assets), we show what would happen if the random component of returns fluctuated between plus and minus one standard deviation. We also show some other realisations. Each of these scenarios show what rational optimising agents would do in the light of a particular realisation for the aggregate shock. (For the idiosyncratic income shocks there are always 100,000 different life histories of realisations). This is a rational expectations solution.

We calculate results for three sets of realizations, all of which have either zero mean or close to it. One has a standard deviation exactly equal to that used by agents but where realizations alternate in sign; one is a random draw from the assumed distribution (a normal). The third is just the mean outcome (zero) in every period. We find that the aggregate results are very similar in all three cases. The reason is similar to that outlined by Krusell and Smith (1998) in explaining their finding that their approximate equilibrium is close to the true solution in an OLG-rational expectations model. This is that most agents can self-insure against some risk quite well by building up a stock of precautionary saving early in life, so mean outcomes for the relevant shock matter a lot but variability is less significant. In the tables of results below we focus on the case where the realized path of shocks has zero mean and the same standard deviation as the true distribution; this realized path of shocks alternates between plus and minus one standard deviation. The charts show outcomes for this and the other realized paths of shocks.

From the set of 100,000 profiles we calculate for each cohort the expected utility of someone about to start their life. We take the actual life histories of the 100,000 in the cohort and calculate the average lifetime utility of this group, a calculation that takes into account both time discounting and survival probabilities. This figure we take to be the ex-ante expected utility of the cohort.

The Simulations

We will show four types of simulation. In the first, state pension generosity is preserved at roughly its current level. We assume that the unfunded pension for someone earning average wages at retirement is worth about 70% of their net-of-contributions final earnings - a figure in line with the current Japanese pension system (see Clark and Mitchell (2002)). In the second set of simulations we assume that the contribution rate (the tax rate) is preserved at roughly its 2001 level of 17.35%. This will require that the typical replacement rate of the state pension decline gradually as the population ages. In the third set of simulations we assume that the government announces now that from 2010 they will gradually reduce the typical replacement rate of unfunded state pensions in a way that leads to a fall in the average replacement rate to just under 15% of net wages (about one fifth its current level) by 2050. In the fourth set of simulations the state pension is phased out almost completely in a gradual way starting in 2010 and ending in 2050[9]. Figure 1 shows how gross replacement rates evolve in the four pension scenarios.

In all simulations we use official (central) Japanese population projections to assess support ratios and calculate contribution rates needed to balance state pension systems[10].

[9] We preserve a tiny unfunded state pension (of 1.5% of average earnings) to prevent zero consumption for some agents that would make numerical solution of the model very difficult.

4. Calibration

The key parameters in the model reflect degrees of risk aversion and the substitutability of consumption over time (that is the inter-temporal substitution of consumption), the rate of pure time preference and the degree of efficiency of financial markets. There is considerable uncertainty about the magnitude of many of these parameters. In the base case we assume a relatively low inter-temporal substitutability of consumption (equal to one third, which implies a coefficient relative risk aversion of 3), we also assume a slightly negative rate of pure time preference, which may seem strange but is consistent with the limited empirical evidence available for Japan. In the absence of bequests, we find that a negative discount rate is needed to generate the level of savings observed in Japan. A negative discount rate is not inconsistent with positive equilibrium real rates of return; see Benninga (1990), and Kocherlakota (1990). Other researchers have used a negative rate of pure time preference to model Japanese household decisions. Kato (1998) calibrates an OLG model with a -7.5% discount rate and Kato (2000) uses the rate of -3.5%. Imrohoroglu, Imrohoroglu and Joines (1999) use a negative discount rate of just under -1% in their numerical simulations. The empirical work of Hurd (1989), based on US data, is also consistent with negative rates of pure time preference. We also show results with a much higher rate of pure preference where agents discount the future at +1.5% a year. This is a very different assumption. With a rate of pure preference of –1.5% an agent at age 30 attaches four and a half times as much weight to outcomes at age 80 as when the rate of pure preference is +1.5%.

The coefficient of risk aversion and the intertemporal elasticity of substitution (which in our specification are the inverse of each other) is also controversial; Cooley and Prescott (1995) use unity for their simulations whereas Auerbach and Kotlikoff (1987) use a coefficient of relative risk aversion of 4, implying the elasticity of substitution is only 0.25. Empirical work by Hansen and Singleton (1983) and Mankiw, Rotemberg and Summers (1985) suggest values a little over unity for intertemporal substitutability implying, in our framework, a coefficient of relative risk aversion a little under unity. Grossman and Shiller (1981), Mankiw (1985) and Hall (1980) found, using US data, values between 0 and 0.4. for the intertemporal elasticity

[10] The source for demographic projections and for projected life expectancy rates used in this paper is the National Institute of Population and Social Security research.

suggesting coefficients of risk aversion well in excess of 2. Hubbard, Skinner and Zeldes (1995) use a relative risk aversion of 3 in their simulations. Zeldes (1989) estimated the risk aversion coefficient as 2.3. Kato (1998) and Kato (2000) use relative risk aversion of 5 and 2.22 respectively in simulation of the Japanese economy. We consider a value of 3 for the risk aversion coefficient is a central estimate but clearly the evidence makes it hard to be confident about what a plausible figure is. We will also show results where the coefficient of risk aversion is set equal to 6. We assume in this simulation that aggregate labour productivity increases at an exogenous rate of 2% a year.

This set of parameters yields an equilibrium path in the stochastic OLG model where the savings rate over the 1990s averages about 20% and the aggregate wealth-income ratio is around 3.3. These are plausible magnitudes for Japan[11].

Income profile

Cross section profiles of Japanese incomes suggest that it is typical for earnings to peak at around age 50 when average earnings are around double the earnings of new workers (20 year olds)[12]. We set the parameters of the earnings process so that on average the income of Japanese workers peaks at the age of 50 when it is roughly double earnings at age 20.

Income volatility

Setting the volatility of the shock to labour income is particularly important for the simulations. As noted above, a significant part of the shocks to individual incomes is likely to be persistent. Hubbard, Skinner and Zeldes (1995) use a model of income dynamics to simulate the impact of social security which is based on characteristics of US household income data. Their model for the log income of household k at time t is the same as specified at (4) above and can be written:

$$y_{kt} = f(age_{kt}) + u_{kt} + \omega_{kt}$$

[11] Ando (2000) reports a ratio of the net worth of Japanese households and non-profit institutions (excluding land) to disposable income for 1998 of 3.26 (Table 1a). He reports an average net saving rate for the period 1970-1998 of 19% (Table 2a).

[12] Japanese Ministry of Health, Labour and Welfare, "Basic Survey on Wage Structure".

$u_{kt} = \phi u_{kt-1} + e_{kt}$

where ω and e are iid shocks that are not correlated and $f(age_{kt})$ is a deterministic function.

A measure of the unconditional volatility of log income is:

$\sigma^2_\omega + \sigma^2_e / (1 - \phi^2)$

Typical values for ϕ, σ_ω and σ_e used by Hubbard et al are 0.955, 0.158 and 0.158. These imply that some income shocks are highly persistent. With these values their measure of the unconditional standard deviation of the shock to log income is 0.56[13].

The dispersion of Japanese wages is lower than in the US. We used information on the distribution of labour incomes of Japanese households over time to find the key parameters of the stochastic process. Data on how the cross-section distribution of incomes for people aged 20 and aged 40 in 1981 evolved over the period to 1996 was used to pick the parameters of equation (4) so as to best match the empirical moments of the distribution. We found that the best fit was achieved when we set the persistence parameter at 0.968 and the standard deviation of the persistent income shocks (e) at 0.1191. The standard deviation of the temporary shock to log incomes was set at 0.076. This generates a cross section standard deviation of log incomes amongst those close to retirement in Japan of around 0.46.

Returns

The historical real returns on NIKKEI 225 index over the past few decades have a mean of around 7-8% with standard deviation of 22-25% depending on the precise period one considers (we have looked at 1960-2000, 1970-2000, 1980-2000). These figures are for gross returns; net of charges annual returns to individuals are likely to be lower by at least 50 basis points, and perhaps by much less.

Returns on bond portfolios are less volatile than on equity portfolios. Miles and Timermann (1999) suggest that a mixed bond and stock portfolio in developed countries would have generated a lower average real return than an equity portfolio

[13] In fact Hubbard et al set different values of ϕ, σ_ω and σ_e for those with no high school, high school and College education. The implied unconditional standard deviation of the shocks to log income for these three groups are 0.64, 0.51 and 0.44 respectively.

and have a significantly lower annual volatility. In Japan such a mixed portfolio might generate an average real return of about 6-7% a year with annual standard deviations of around 17.5%. Stock returns with these characteristics have been typical in many developed countries in the past. (See Dimson, Marsh and Staunton (2003)). The above figures are before any deductions for charges, and for this reason we think of net returns on risky assets with a mean of 6.5% and volatility of 17.5% as relatively optimistic.

5. Results

There are four different reform scenarios. Figure 1 shows the average replacement rate at retirement of the unfunded, state pension in each. Figure 2 shows the path of the contribution rate that balances the PAYGO system in each case. The contribution rate needed to balance the system with a flat net replacement rate needs to rise from about 17.4% to about 27%. The rise is not steady since we allow for the increase in the retirement age that will take place around 20 years from now which temporarily allows the contribution rate to fall slightly. A decision to keep the contribution rate at its current level (17.4%) requires that ultimately the replacement rate would need to fall from around 70% of incomes to just over 40% by 2050. A decision to phase down dramatically the replacement rate to just under 15% would allow the contribution rate needed to balance the system to fall to about 7% - well under half its current level.

Using these different paths for the contribution rates and for the value of unfunded, state pensions we then undertake simulations with 100,000 individuals *of each and every* cohort followed through their lives. Both the aggregate outcomes and, of course, the individual outcomes, depend on the realization of shocks. The income shocks are idiosyncratic and so tend to get averaged out for the aggregate outcomes. But the rate of return shocks are common financial market shocks. So when we undertake dynamic stochastic simulations we need to make some common assumptions on the realization of rate of return shocks to be able to compare different simulations with different pension arrangements.

We consider 3 different realizations for the stochastic element of the return on risky assets.

1. the stochastic element comes from one draw from an iid normal for the time series from 1900 to 2150.
2. the stochastic element of returns alternate each year between being one standard deviation above the mean, and one standard deviation between the mean.
3. the shock is zero in all periods

The three different paths used to compute the outcomes are just three particular realizations for the aggregate shock. The broad nature of the results turns out to be similar for each of the three realizations.

Table 1 and figures 3 and 4 summarise the main macroeconomic implications of the simulations when we assume that $\beta = 0.5$. Table 3 focuses on aggregate outcomes when we assume there is an absence of annuities ($\beta = 0$). Tables 2 and 4 focus on welfare measures for the various pension scenarios under the two different assumptions about annuity market efficiency. In the tables we report results where the realization of the rate of return shocks for the risky assets alternate between +1 and –1 standard deviation[14].

Tables 1 and 3 show that the aggregate savings rate is projected to move sharply as a result of ageing. How great this effect is depends very much on pension reform. If unfunded pensions remain, on average, worth about 70% of net wages the savings rate is predicted to fall sharply – from around 21% in 2000 to about 13% by 2060, if β=0.5, and to about 15% in the absence of annuities. If the contribution rate to balance an unfunded system is kept at 17.4% the decline in the saving rate is much more gentle. The saving rate in 2060 is around 17% with annuities and about 20% without annuities. If the state pension is scaled back dramatically so it becomes a small fraction of wealth at retirement (worth on average just under 15% of net salary at retirement) the aggregate saving rate is roughly flat if there are annuities and rises significantly if there are no annuities. If state pensions are phased out almost completely the aggregate savings rate moves *up* sharply over the long term when there are no annuities and rises slightly if there are annuities.

[14] We show in figures 3and 4 outcomes for the other paths of realized rate of return shocks also.

Figure 3 shows that the aggregate savings rate is slightly influenced by the choice of ex-post realizations of shocks to rates of return on risky assets. But it is much more influenced by the nature of pension reform. The four panels in figure 3 show the evolution of savings under the four different pension scenarios. (In this figure we take $\beta = 0.5$).

Not surprisingly when there is either a partial or almost complete switch to funding the aggregate stock of wealth is higher (Tables 1 and 3). With a constant contribution rate to the state pension system the stock of aggregate assets by 2060 is between 42% and 54% higher than it would be with a constant replacement rate. The stock of wealth would be more than twice as high by 2060 if state pensions were to be phased out completely. These big effects come through with or without annuities.

Figure 4 shows the evolution of the share of aggregate wealth held in risky assets. This is highly sensitive to pension arrangements. If households have to rely for a much greater share of their retirement consumption on their own funds, and less on state pensions that are independent of financial market risk, they react by switching a large part of their portfolios away from risky assets and into safe assets. The greater is reliance upon funding, and the lower are PAYGO pensions, the larger is the share of wealth that is invested in safe assets. The scale of the effect is very large. With pensions worth, on average, 70% of net final salary usually more than 90% of wealth is invested in risky assets. If the net replacement rate falls to just under 15% that share eventually falls to around 70%. If state pensions are phased out entirely the share of risky assets falls to around 60% by 2060. This result is not much affected by the absence of annuities contracts.

Transition towards greater reliance upon funding requires a period when aggregate consumption falls below its path under a policy of preserving the generosity of unfunded pensions. But ultimately aggregate consumption rises significantly above that level when funding becomes more important. How great the decline in consumption is, how long it takes before consumption then overtakes the path under a policy of no reform, and by how much consumption in the long term can be higher depend both upon how great the move to funding is and is somewhat affected by the realizations of shocks to rates of return.

The paths for aggregate consumption suggest that some generations might be worse off as a result of a move to funding. In particular, cohorts who are relatively early in the working lives at the initiation of reform find that they need to continue paying substantial contributions to finance PAYGO pensions to their parents generation but will receive a relatively small pension by the time they retire 30 or 40 years hence.

It is what happens to the *welfare* of agents of different generations that really matters. Tables 2 and 4 shows in some detail how agents of different ages are affected by various reform strategies with and without annuities. We take as the base case a situation that we have called "no reform" - PAYGO pensions continue to be paid at a generous level (assumed to average around 70% of net wages). The tables gives an indication of the effect upon the lifetime utility of people of different ages of either pegging the contribution rate at its current level or scaling back pensions sharply from 2010. We measure the welfare implications of phasing out pensions by calculating a measure of the expected utility of agents of various cohorts (from those who are aged 60 today to those who will not be born for another 50 years). We compare how each cohort does in the base case where the state pension stays constant (relative to average earnings) with how they do when the ratio of the state pension to average earnings starts to fall as a move is made towards funding. In all cases we look at a scenario where the realization of the random component of the return on risky assets turns out to alternate between +1 and -1 standards deviation.

The first column for each reform strategy shows whether those in the relevant age cohort gain (G) or lose (L). The second column is an estimate of the scale of the gain or loss; it is the percentage by which consumption in the base case simulation would need to have been higher or lower to generate the same level of welfare as is given by the transition path. This is the compensating variation in consumption. So for people aged 50 in 2002 (born in 1952) the average decline in utility generated by pegging the contribution rate is the equivalent of a 2% cut in lifetime consumption.

Under a strategy of dramatically scaling back the level of PAYGO pension benefits more of those currently alive lose out than if the contribution rate is pegged. But the long-run gains are very much higher. In the absence of annuities the losses from sharply reducing state pensions for those generations who do lose out are slightly greater than when annuities (albeit at less than actuarially fair rates) are available. More or less completely phasing out state pensions generates a welfare loss to the cohort born in 1982 of the equivalent of 7% of consumption when there are no annuities and a loss of around 5% when there are annuities. But the welfare gains for those born in the twenty first century are much larger.

The result that a large proportion of those alive now would be worse off if the unfunded state scheme is phased out - even though *every* future generation is much better off - illustrates the nature of the transition problem rather clearly. Democratically elected governments facing voters who focus on the direct implications to them (and not to all future generations) of changes to state pension systems would find it hard to get support for this kind of transition plan. Tables 2 and 4 suggest that once a transition from an unfunded to a funded scheme is complete welfare for *all* subsequent generations will be higher, but without relying on deficit financing the transition will cause certain generations to be worse off, and those generations could form a majority of voters thus permanently blocking any change.

The losses of the transition generations are small relative to the gains of the future generations but they are most definitely non-trivial. The tables suggest that there is unlikely to be a painless (i.e. Pareto improving) way of achieving reform.

It is important to stress that these welfare calculations are all about *expected utility at birth.* The fact that expected welfare for an agent born in, say, 2012 is higher when there has been a substantial movement towards funding and a cut in the generosity of unfunded pensions does not tell us that all individuals born then will turn out to be better off under a reform strategy that scales back PAYGO pensions. What it does tell us is that someone born then who understand the risks they faced *and did not know the realizations of income and rate of return shocks* would judge, at the start of their life, that they are better off in a world with more funding and less reliance upon PAYGO pensions. There will be individuals who turn out subsequently to be worse off with more funded pensions. One way in which we can assess what the distribution of gainers and losers (ex-post) looks like is to compare the distribution of retirement consumption and of ex-post utility for different cohorts under different pension regimes. This is what we consider in the next section.

6: The distribution of retirement consumption and of lifetime utility under different pension arrangements.

Different pension systems generate different distributions of retirement consumption and of ex-post lifetime utility. Here we focus on the spread of retirement consumption and of lifetime utility for different cohorts under different pension scenarios. We consider 4 cohorts: those aged 60 in 2002; those aged 35 in 2002; those born in 2002 (who will not start work until 2022); and those born in 2030 who will not start work until 2050. We consider the distribution of consumption at age 65 and of lifetime utility for members of each of these cohorts under the 4 different pension regimes. For each pension regime we report results on the distribution of outcomes where return shocks alternate between +1 and –1 standard deviation[15]. Table 5 looks at the distribution of ex-post utility of various cohorts. There are four pension regimes, as before: a regime in which unfunded, PAYGO pensions on average remain worth 70% of net final salary; a regime with a constant contribution rate of 17.4%; a regime in which PAYGO pensions by 2050 are worth on average just under 15% of net final salary; and a scenario in which state pensions are almost completely phased out. The tables show results for β=0.5.

For those born in 2030 (and whose age 65 consumption comes in 2095) nearly everyone has a higher level of consumption *at retirement* and significantly higher lifetime utility if there has been a move towards greater reliance upon funding. But at the very bottom of the income distribution there are losers, as least as regards consumption. The consumption of the household in this cohort at the lowest percentile is consistently lower the greater is reliance upon private, funded (DC) pensions. This is because the PAYGO scheme is redistributive and if it is phased out those at the very bottom of the income may have lower consumption at retirement. Even so they may still gain in utility terms because contribution rates are lower and such low income households are often credit restricted early in life and may gain from paying less in compulsory contributions. Indeed focusing on expected utility there are gainers right across the distribution for the cohort born in 2030 from a scaling back in state pensions.

For the cohorts who do not live through a transition towards a more funded system (and therefore do not face the double payment burden) almost everyone is better off at retirement. But as we consider cohorts born earlier, and who to different extents do live through the transition, the position is different. Consider the cohort born in 1967, who enter work in 1987. Right across the distribution of retirement resources we find that this cohort are worse off the greater is the scale of the move towards funding. They enter retirement in year 2032. At that point consumption is lower right across the distribution than it would be if PAYGO pension generosity had been left unchanged. The scale of the loss in consumption at retirement is substantial and is greater for those at the bottom of the income distribution. With a complete phasing out of pensions for this cohort utility falls by the equivalent of a cut in lifetime consumption of about 2.5% for someone at the middle of the income distribution, but by about 4.5% for someone at the bottom.

The important point to emerge from this distributional analysis is that in the longer run pretty much everyone gains from a switch to funding - even those with very low lifetime resources. But for those who face the double payment burden (at least for the fairly simple transitional arrangement simulated here) the losses are also very hard to avoid – right across the income distribution there is lower consumption at retirement. This is especially marked for those in their mid thirties today. We find that without annuities the scale of losses for those that are worse off when state pension are reduced are somewhat greater.

7: Alternative Parameterisations:

In this section we consider how the simulation results are affected by changes in two key parameters. We consider three alternatives: first we take a much higher rate of risk aversion, increasing the coefficient of relative aversion from 3 to 6. Second we

[15] Results are not very different for the two other sets of simulations with alternative realized paths for

take a much higher rate of time preference, raising the discount rate from –1.5% to +1.5%.

Tables 7 and 8 briefly summarise the results. In all cases we consider the case where there are (imperfect) annuities available. In constructing these figures we once again have calculated optimal decisions for individuals based on draws for income and rate of return shocks. For each cohort we then aggregate over 100,000 agents and show the average results for each regime, at each point and for a given evolution of the random rate of return. In these tables we show results where the random component of the rate of return alternates between +1 and –1 standard deviation. As noted above this is just a particular realization for the shock to the risky rate of return (alternative paths did not have a substantial impact upon aggregate outcomes).

Table 7 summarises the aggregate results when the coefficient of risk aversion is much higher. Comparing those results with those in table 1 reveals that with higher risk aversion a very substantially smaller share of wealth is invested in risky assets. When state pensions are reduced very sharply the share of risky assets in portfolios by 2060 falls to only around 33% with high risk aversion as against a figure of around 60% with a coefficient of risk aversion of 3. The saving rate is also consistently lower with high risk aversion. The incentive to accumulate wealth is pulled in two directions by greater risk aversion. On the one hand there is a tendency for savings to rise because of greater precautionary demands. But with risky assets so much less attractive, the overall desirability of saving is actually lower. The aggregate wealth to income ratio is significantly lower when there is more risk aversion.

But the key welfare conclusion from the analysis is little changed with higher risk aversion. Agents of working age now tend to lose out slightly from a decision to move towards funding. Those born in the future stand to gain, and their gain is greater than the loss of the working generations that live through the transition. National Income is much higher in the long run when pensions are scaled back.

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the return shocks. The reason is that all the paths have shocks that average out to zero.

We also performed simulations with a much lower rate of risk aversion. When the coefficient of risk aversion was set to unity (log preferences) we consistently found that the rate of saving and the capital to labour ratio was implausibly high. The problem here is that lowering the rate of risk aversion increases the degree of intertemporal substitutability generating much greater saving for plausible values of rates of return.

Table 8 shows aggregate outcomes when the rate of time preference is +1.5%. Not surprisingly the saving rate is consistently very much lower than in Table 1 where the rate of time preference was *minus* 1.5%. (The reason for using a negative rate of preference was largely to try to match the relatively high Japanese saving rate). As a result the aggregate wealth to income ratio is much lower with positive time preference. But once again savings rates and wealth to income ratios are ultimately boosted very significantly by a move towards funding of pensions and a decline in the generosity of unfunded pensions. The overall pattern of the welfare implications of reforms is little affected by assumptions about the discount rate. But the long run gains are greater with a much higher rate of time preference. The reason is straightforward. A major effect of phasing out unfunded, state pensions is that individuals do not need to make significant compulsory contributions from labour income. This is very valuable early in life to agents facing credit restrictions. The scale of those credit restrictions is greater the more impatient consumers are. It is important to note that a significant part of the gain from a switch towards giving people discretion about the scale and timing of contributions towards their own personal pension pot is the value of the flexibility this gives – something which is absent in most unfunded social security systems where contributions are typically a given proportion of earnings.

8 Conclusions:

Our key findings are these.

The overall saving rate, and particularly the aggregate stock of assets, is likely to be *highly* sensitive to the generosity of unfunded state pensions. It is likely that a long-run implication of a switch to much greater reliance upon funded pensions is that consumption and welfare for future generations will be significantly higher. This result is true even when annuities markets do not work well. Yet it is likely to be hard to engineer a transition to much greater reliance upon funding without leaving a substantial proportion of today's adults at least slightly worse off.

There is a powerful link between overall portfolio allocation and the pension system. The more generous are state, unfunded pensions –the value of which is only indirectly (and perhaps weakly) linked to rates of return on financial assets - the greater is the proportion of wealth invested in risky assets. If people come to rely heavily upon funded pensions that expose their retirement resources to financial market risk then they are likely to respond by holding a large share of their financial wealth in safe assets.

It is important to note that a significant part of the gain of a switch towards funding stems from giving people discretion about the scale and timing of contributions towards their own personal pension pot. There is a significant value to the flexibility this brings – something which is absent in most unfunded social security systems where contributions are typically a given proportion of earnings. One important implication of this is that there may be substantial gains to be had by increasing the flexibility in the timing of contributions within an existing unfunded, PAYGO pension system.

Longer run gains from a switch towards greater reliance upon funding, and away from an unfunded system where pensions are linked to salaries, do not overwhelmingly go to the better off. But those at the bottom of the income distribution do least well when the re-distributive PAYGO system is scaled back.

These results are broadly in line with those reported by Kotlikoff, Smetters and Walliser (1999) and by De Nardi, Imrohoroglu and Sargent (1999). Both papers report very substantial long run welfare gains to scaling back the generosity of PAYGO pensions and very large increases in the capital stock. In their papers there is no rate of return uncertainty, no focus on financial market efficiency and the models are calibrated to the United States. It is significant that the main results carry over to a model with more emphasis on uncertainty and financial market efficiency and where the model is calibrated to the, very different, Japanese economy.

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Table 1: Aggregate Outcomes, Annuity Parameter 0.5

	Keep replacement rate of state pension at 72%	Keep contribution rate constant at 17.4%	Phase down replacement rate to 14.5% by 2050	Phase down replacement rate to 1.4% by 2050
Savings Rate				
2000	21.1%	22.3%	21.3%	21.5%
2020	15.5%	20.6%	20.3%	21.9%
2040	14.5%	18.9%	23.8%	26.5%
2060	12.9%	16.8%	21.9%	24.0%
Wealth-Income Ratio				
2000	3.8	3.9	3.8	3.8
2020	4.2	4.6	4.3	4.4
2040	4.2	5.1	5.3	5.7
2060	4.0	5.3	6.5	7.4
Aggregate Assets				
2000	13.8	14.2	13.8	13.8
2020	20.5	24.1	22.2	22.8
2040	28.9	37.4	41.1	45.3
2060	38.8	55.2	72.2	83.1
Share of Risky Assets				
2000	94.8%	95.5%	95.1%	95.2%
2020	93.6%	89.6%	94.2%	94.1%
2040	93.9%	86.4%	82.5%	77.0%
2060	94.3%	83.5%	70.1%	61.7%
GDP relative to base				
2000		0.9%	0.0%	0.0%
2010		4.0%	0.7%	1.0%
2020		6.2%	4.8%	6.2%
2030		4.5%	8.6%	11.0%
2040		5.7%	11.1%	13.9%
2050		6.9%	14.0%	17.1%
2060		6.8%	12.8%	15.2%

Notes:

Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean. In these simulations annuity market efficiency (β) is set to 0.5. The coefficient of risk aversion is set at 3.0 and the rate of pure time preference is -1.5% p.a. State, unfunded pensions are related to final salary.

Table 2: Gainers and Losers from Pension reform, Annuity Parameter β = 0.5

Compensating Variation in Consumption – equivalent variation in lifetime consumption to keep utility at base level: gains (+) or losses (-)

Cohort age in 2002 (and year when born)	Keep contribution rate constant at 17.4%			Phase down replacement rate to 14.5% by 2050			Phase down replacement rate to 1.4% by 2050	
60 (1942)	L	-0.2%		G	0.4%		G	0.4%
50 (1952)	L	-2.0%		L	-0.2%		L	-0.2%
40 (1962)	L	-1.6%		L	-1.6%		L	-2.1%
30 (1972)	G	0.4%		L	-2.8%		L	-3.9%
20 (1982)	G	1.4%		L	-3.7%		L	-5.9%
10 (1992)	G	3.7%		L	-1.6%		L	-3.9%
0 (2002)	G	4.6%		G	3.2%		G	2.2%
-10 (2012)	G	6.1%		G	9.2%		G	10.1%
-20 (2022)	G	7.7%		G	13.4%		G	15.6%
-30 (2032)	G	8.0%		G	14.8%		G	17.4%
-40 (2042)	G	8.0%		G	14.8%		G	17.4%
-50 (2052)	G	8.0%		G	14.8%		G	17.4%

G = gain; L = lose

Notes:

Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean. In these simulations annuity market efficiency (β) is set to 0.5. The coefficient of risk aversion is set at 3.0 and the rate of pure time preference is -1.5% p.a.. State, unfunded pensions are related to final salary.

Table 3: Aggregate Outcomes, No Annuities

	Keep replacement rate of state pension at 72%	Keep contribution rate constant at 17.4%	Phase down replacement rate to 14.5% by 2050	Phase down replacement rate to 1.4% by 2050
Savings Rate				
2000	23%	24%	23%	23%
2020	18%	23%	23%	25%
2040	17%	22%	28%	32%
2060	15%	20%	27%	30%
Wealth-Income ratio				
2000	3.9	3.9	3.8	3.8
2020	4.2	4.7	4.4	4.4
2040	4.2	5.3	5.6	6.1
2060	3.9	5.5	7.0	8.3
Aggregate Assets				
2000	14.1	14.5	14.0	14.0
2020	20.8	25.1	22.6	23.5
2040	28.9	39.7	44.2	50.1
2060	38.5	59.3	81.9	100.0
Share of Risky Assets				
2000	97%	97%	97%	97%
2020	96%	92%	96%	96%
2040	97%	90%	85%	79%
2060	97%	87%	72%	62%
GDP relative to base				
2000		0.9%	-0.2%	-0.2%
2010		4.3%	0.6%	0.9%
2020		7.1%	4.9%	6.6%
2030		6.1%	9.7%	12.8%
2040		7.9%	13.8%	17.7%
2050		9.5%	18.3%	22.8%
2060		9.8%	18.2%	23.0%

Notes:

Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean.
In these simulations annuity market efficiency (β) is set to 0. The coefficient of risk aversion is set at 3.0 and the rate of pure time preference is -1.5% p.a.. State, unfunded pensions are related to final salary.

Table 4: Gainers and Losers from Pension reform, No Annuities

Compensating Variation in Consumption – equivalent variation in lifetime consumption to keep utility at base level: gains (+) or losses (-)

Cohort age in 2002 (and year when born)	Keep contribution rate constant at 17.4%			Phase down replacement rate to 14.5% by 2050			Phase down replacement rate to 1.4% by 2050	
60 (1942)	L	-0.3%		G	0.5%		G	0.6%
50 (1952)	L	-2.2%		L	-0.1%		L	-0.1%
40 (1962)	L	-1.8%		L	-1.7%		L	-2.2%
30 (1972)	G	0.2%		L	-3.0%		L	-4.1%
20 (1982)	G	1.1%		L	-4.0%		L	-6.9%
10 (1992)	G	3.4%		L	-2.3%		L	-5.1%
0 (2002)	G	4.2%		G	2.5%		G	0.9%
-10 (2012)	G	5.7%		G	8.2%		G	7.9%
-20 (2022)	G	7.3%		G	12.4%		G	14.0%
-30 (2032)	G	7.6%		G	13.8%		G	15.7%
-40 (2042)	G	7.6%		G	13.8%		G	15.7%
-50 (2052)	G	7.6%		G	13.8%		G	15.6%

G = gain; L = lose

Notes:

Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean.
In these simulations annuity market efficiency (β) is set to 0. The coefficient of risk aversion is set at 3.0 and the rate of pure time preference is -1.5% p.a.. State, unfunded pensions are related to final salary.

Table 5: Ex-post Utility of agents born in 1942, 1967, 2002 and 2030

Percentile of Distribution	Keep contribution rate constant at 17.4%	Phase down replacement rate to 14.5% by 2050	Phase down replacement rate to 1.4% by 2050
1942 cohort			
1%	-0.4%	0.5%	0.7%
5%	-0.3%	0.4%	0.5%
20%	-0.2%	0.3%	0.4%
50%	-0.2%	0.3%	0.4%
75%	-0.2%	0.3%	0.4%
95%	-0.2%	0.3%	0.4%
1967 cohort			
1%	-1.9%	-3.5%	-4.5%
5%	-1.2%	-2.6%	-3.3%
20%	-0.6%	-2.0%	-2.6%
50%	-0.5%	-1.8%	-2.5%
75%	-0.6%	-1.9%	-2.5%
95%	-0.5%	-1.8%	-2.4%
2002 cohort			
1%	2.4%	0.1%	-3.4%
5%	4.0%	2.1%	0.1%
20%	4.8%	3.2%	2.2%
50%	5.0%	3.9%	3.6%
75%	5.1%	4.4%	4.4%
95%	5.2%	4.9%	5.3%
2030 cohort			
1%	5.9%	10.3%	10.3%
5%	7.4%	13.1%	14.6%
20%	8.2%	14.8%	17.3%
50%	8.4%	16.0%	19.3%
75%	8.5%	16.5%	20.3%
95%	8.6%	17.1%	21.3%

Notes:

Figures show level of utility for member of cohort at given percentiles of utility distribution. “% gain” is the equivalent change in lifetime consumption to generate the change in utility relative to the base case of a constant net replacement rate for the state pension. This is the compensating variation in consumption.

Rates of return alternate between being one standard deviation above and one standard deviation below the average.

Table 6: Ex-post Consumption at age 65 of agents born in 1942, 1967, 2002 and 2030

Percentile of Distribution	Keep contribution rate constant at 17.4%	Phase down replacement rate to 14.5% by 2050	Phase down replacement rate to 1.4% by 2050
1942 cohort			
1%	-1.2%	1.1%	1.4%
5%	-0.8%	0.9%	1.1%
20%	-0.6%	0.8%	1.0%
50%	-0.5%	0.8%	0.9%
75%	-0.5%	0.8%	1.0%
95%	-0.5%	0.8%	1.0%
1967 cohort			
1%	-7.3%	-9.6%	-12.0%
5%	-4.7%	-7.0%	-8.7%
20%	-2.9%	-5.0%	-6.5%
50%	-2.0%	-4.2%	-5.6%
75%	-1.7%	-3.9%	-5.2%
95%	-1.3%	-3.5%	-4.6%
2002 cohort			
1%	-7.3%	-17.3%	-25.3%
5%	-3.4%	-11.5%	-17.8%
20%	-0.9%	-6.8%	-11.4%
50%	0.2%	-3.5%	-6.5%
75%	0.6%	-1.5%	-3.6%
95%	1.3%	0.8%	-0.3%
2030 cohort			
1%	-6.1%	-12.8%	-18.2%
5%	-1.8%	-5.7%	-9.7%
20%	0.8%	-0.6%	-2.6%
50%	2.2%	3.3%	2.6%
75%	2.8%	5.3%	5.5%
95%	3.6%	7.6%	8.6%

Notes:

Figures show level of consumption at age 65 for member of cohort at given percentiles of age 65 consumption.

Rates of return alternate between being one standard deviation above and one standard deviation below the average.

Table 7: Stochastic Simulations: Coefficient of Risk Aversion = 6

	Keep replacement rate of state pension at 72%	Keep contribution rate constant at 17.4%	Phase down replacement rate to 14.5% by 2050	Phase down replacement rate to 1.4% by 2050
Savings Rate				
2000	15%	16%	15%	15%
2020	9%	15%	15%	17%
2040	9%	12%	18%	21%
2060	7%	10%	16%	18%
Wealth-Income Ratio				
2000	3.3	3.4	3.2	3.2
2020	3.6	4.2	3.8	3.9
2040	3.6	4.8	5.2	5.7
2060	3.4	5.0	6.5	7.5
Aggregate Assets				
2000	10.8	11.2	10.6	10.6
2020	15.8	19.5	17.6	18.3
2040	22.4	31.0	35.1	39.5
2060	29.8	45.9	63.3	74.4
Share of Risky Assets				
2000	67.7%	68.1%	69.0%	69.3%
2020	65.9%	58.4%	65.2%	64.6%
2040	65.8%	53.3%	48.3%	43.4%
2060	66.9%	51.2%	39.9%	33.7%
GDP relative to base				
2000		0.8%	0.0%	0.0%
2010		3.7%	0.8%	1.1%
2020		5.5%	4.7%	6.1%
2030		3.0%	7.5%	9.6%
2040		3.9%	9.3%	11.8%
2050		4.9%	11.9%	14.9%
2060		4.5%	9.8%	12.1%

Notes:
Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean.
In these simulations annuity market efficiency is set to 0.5. The coefficient of risk aversion is set at 6.0 and the rate of pure time preference is -1.5% p.a.. State, unfunded pensions are related to final salary.

Table 8: Stochastic Simulation Rate of pre time preference +1.5%

	Keep replacement rate of state pension at 72%	Keep contribution rate constant at 17.4%	Phase down replacement rate to 14.5% by 2050	Phase down replacement rate to 1.4% by 2050
Savings Rate				
2000	13%	14%	13%	13%
2020	9%	14%	13%	15%
2040	9%	13%	18%	21%
2060	7%	11%	15%	17%
Wealth-Income Ratio				
2000	2.4	2.4	2.4	2.4
2020	2.6	3.1	2.7	2.8
2040	2.6	3.6	3.9	4.3
2060	2.4	3.8	5.2	6.2
Aggregate Assets				
2000	7.8	8.0	7.7	7.7
2020	11.3	14.3	12.4	12.9
2040	16.1	23.5	26.8	30.9
2060	21.1	35.3	51.7	64.1
Share of Risky Assets				
2000	97.9%	98.3%	98.0%	98.0%
2020	97.3%	94.3%	97.7%	97.6%
2040	97.7%	91.8%	87.3%	80.7%
2060	97.9%	88.6%	71.5%	60.5%
GDP relative to base				
2000		0.6%	-0.1%	-0.1%
2010		4.2%	0.4%	0.6%
2020		6.9%	4.6%	6.0%
2030		5.2%	9.1%	11.8%
2040		6.7%	12.9%	16.3%
2050		8.0%	16.5%	20.4%
2060		7.8%	15.0%	18.6%

Notes:

Figures show the simulation results when shocks alternate between one standard deviation above the mean and one standard deviation below the mean.
In these simulations annuity market efficiency is set to 0.5. The coefficient of risk aversion is set at 3.0 and the rate of pure time preference is +1.5% p.a.. State, unfunded pensions are related to final salary.

Figure 1: Replacement Rates For State Pensions in Different Scenarios

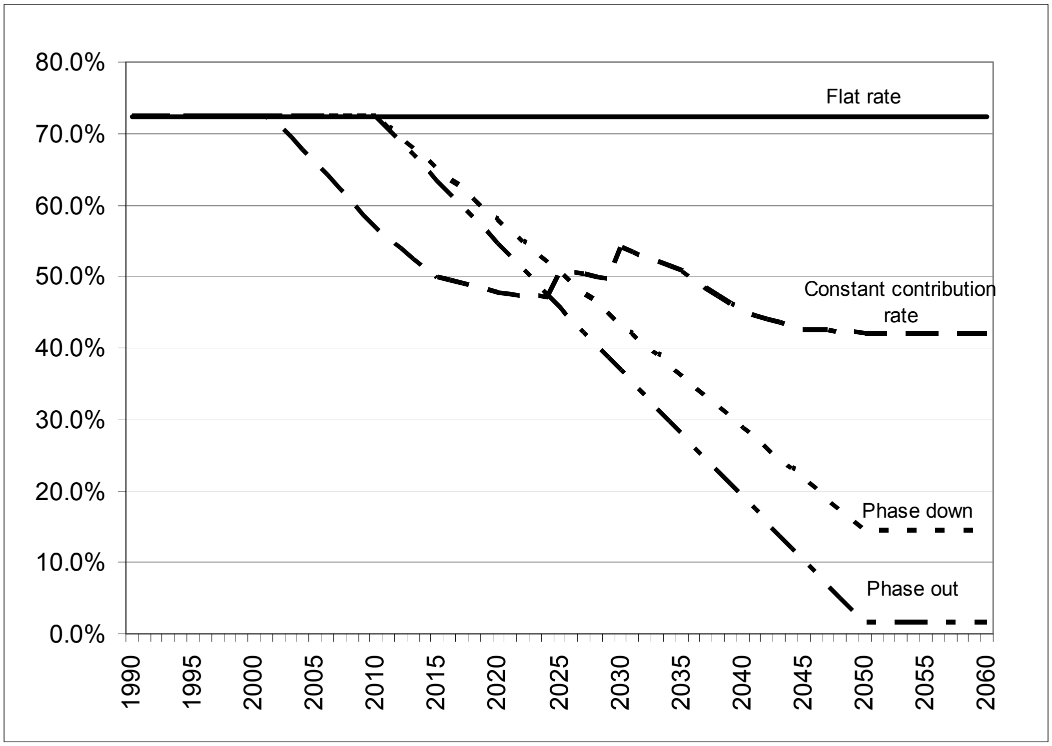


Figure 2: Contribution Rates To Balance Unfunded State Pensions

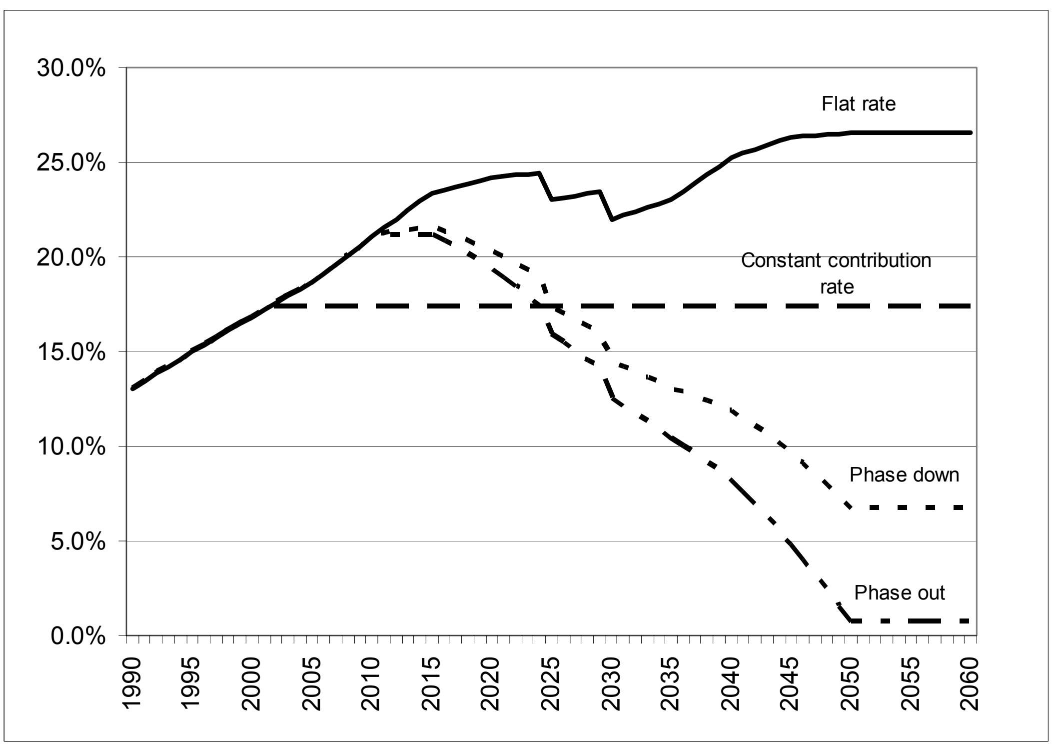
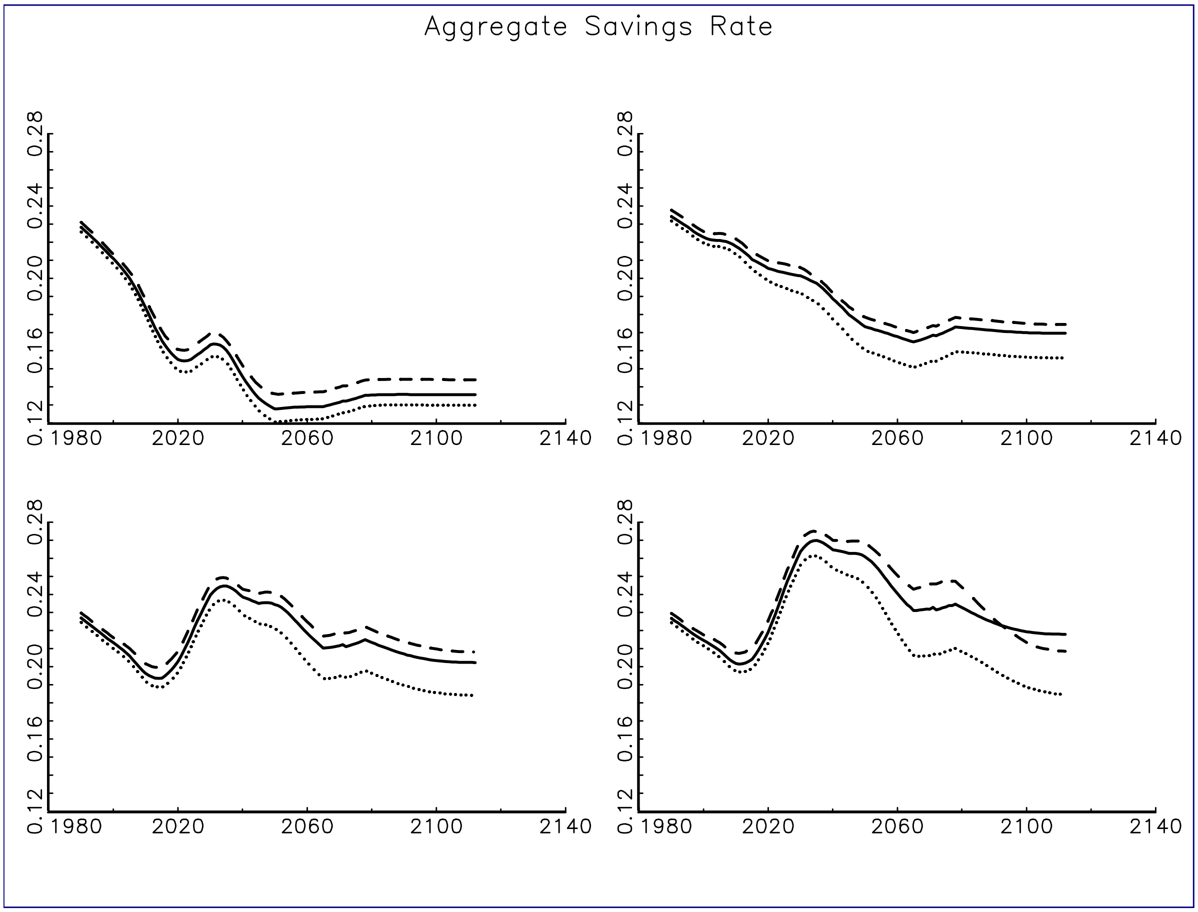
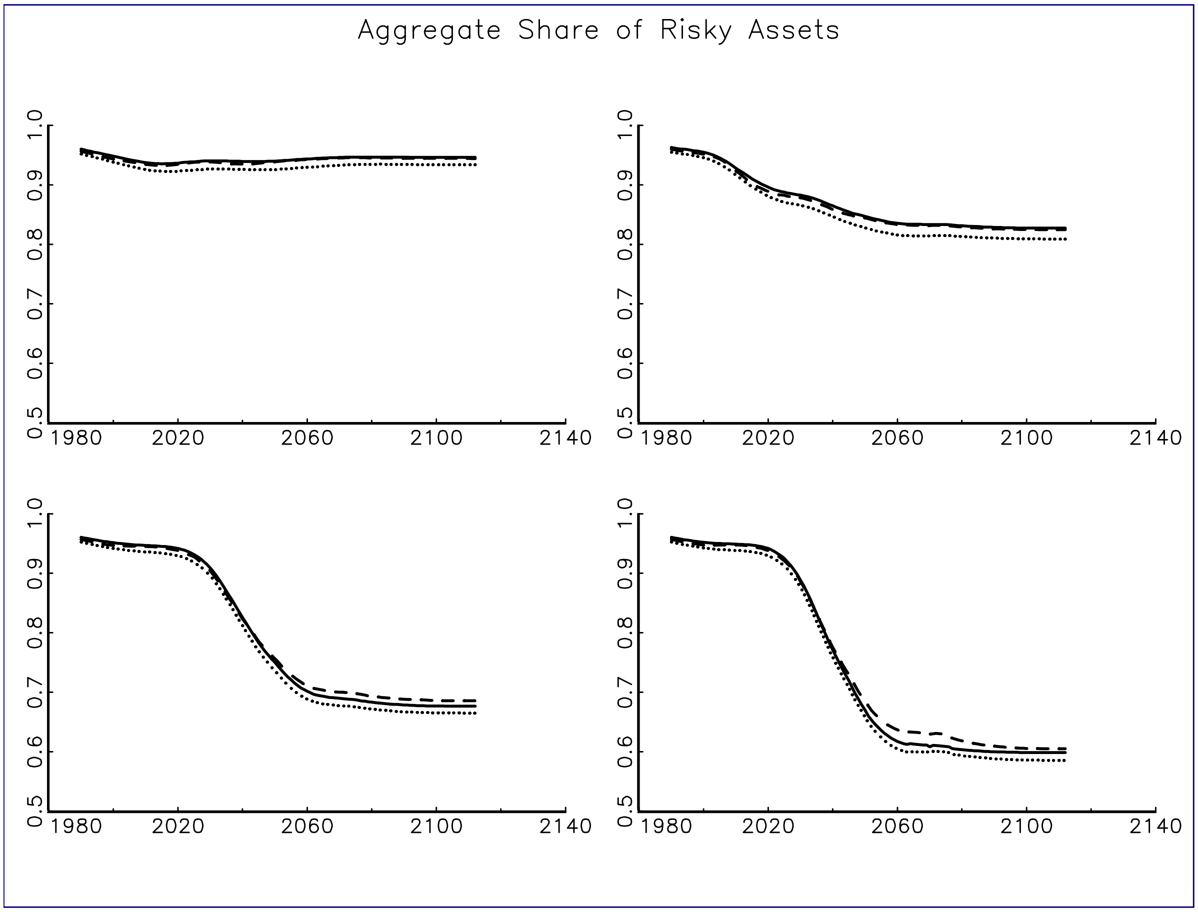


Figure 3:



[1] The four panels in figures 3 and 4 correspond to four different pension scenarios. Left to right, they are i) preserve the replacement rate at the 2003 level ii) scale back pensions so that balanced budget contribution rates are flat; iii) reduce the average replacement rate down to one fifth of its 2003 level by 2050 iv) eliminate PAYG pension almost completely by 2050. Within each panel we show 3 paths for the realizations of the stochastic part of the rate of return: the solid line shows outcomes when rate of return shocks turn out to be zero; the dotted line show outcomes when we alternate between positive shocks of one standard deviation and negative shocks of one standard deviation; the dashed line show outcomes where we have a single draw of randomly generated IID shocks.
All panels show the case where $\beta = 0.5$

Figure 4 :



Aggregate Share of Risky Assets
0.5
0.6
0.7
0.8
0.9
1.0
1980
2020
2060
2100
2140